Bayesian Estimation of Simultaneous Equation Models with Outliers and Multicollinearity Problem

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ABSTRACT

Outliers and multicollinearity are problems in the analysis of Simultaneous Equation Model (SEM) especially in applied research. They can lead to bias or inefficiency estimators. This study employed a Bayesian technique for estimation of SEM that is characterized by both multicollinearity and outliers. Monte Carlo experiment was applied while the data sets with specified outliers and multicollinearity were simulated for the SEM. The estimates of Bayesian and classical methods namely: two stage Least Squares (2SLS), Three Stage Least Squares (3SLS), Limited Information Maximum Likelihood (LIML) and Ordinary Least Squares (OLS) in simultaneous equation model were then compared. The criteria used for comparison were the Mean Square Error (MSE) and Absolute Bias (ABIAS). The Bayesian method of estimation outperformed classical methods in terms of MSE and ABIAS. However, the classical method has the same performance with Bayesian method when there are no outliers and multicollinearity in the simultaneous equation model. Hence, Bayesian method of estimation is preferred than classical method when there is problem of outlier and multicollinearity in a just identified simultaneous equation model.

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1. INTRODUCTION

Researchers often face the problems of multicollinearity and outliers in applied works either planned or not planned. Multicollinearity and outliers can lead to poor predictive power of the model and statistical inferences. These can also lead to inefficiency of estimators of simultaneous equation model. When multicollinearity occurs in the simultaneous equation model, so many classical simultaneous equation estimators will be difficult to apply, especially the two-stage least squares method (Ozbay and Toker, 2018). Outliers are typical observations that are greatly different from group of observations. They can also make the models to have high error rates and substantial distortions for both parameter and statistic estimates (Zimmerman, 1998).

In recent times, attention has been given to the problem of outliers and multicollinearity in both classical and Bayesian econometrics especially in regression models (Duzan and Shariff 2015; Shariff and Ferdaos 2017; Adepoju and Ojo, 2018; Ojo, 2020 & Oyewole 2022). There is not so much research on outliers in simultaneous equation models but there are limited researches on multicollinearity in simultaneous equation models. Major works carried out on the outliers in simultaneous equation models are Mishra (2008) and Adepoju & Olaomi (2012) while researches on multicollinearity in simultaneous equation models are Schink and Chiu (1996), Agunbiade and Iyaniwura (2010), Agunbiade (2011), Mishra (2017), Ozbay and Toker (2018).

Mishra (2008) proposed a robust method that generalizes the two stage Least Squares (2SLS) to the Weighted Two Stage Least Squares (W2SLS) to tackle the effect of outliers and perturbations in data matrix. Monte Carlo method experiment was conducted to examine the performance of the proposed method in simultaneous equations. It was found out that the robustness of the proposed method did not disrupt the magnitude of outliers but sensitive to the number of outliers in the data matrix.

The performance of five estimators; Ordinary Least Squares (OLS), Two Stage Least Squares (2SLS), Three Stage Least Squares (3SLS), Generalized Method of Moment (GMM), and Weighted Two Stage Least Squares (W2SLS) of simultaneous equations model parameters with first order autocorrelation levels of error terms and there is outliers in the data at small sample sizes were considered by Adepoju and Olaomi (2012). It was observed that the system method performed better than the single equation for all the cases of outliers considered.

Agunbiade and Iyaniwura (2010) investigated the performance of six different estimation techniques of a just identified simultaneous three-equation model with three multi-collinear exogenous variables. The estimators considered under the three levels of multicollinearity were Ordinary Least Squares (OLS), Two Stage Least Squares (2SLS), Three Stage Least Squares (3SLS), Limited Information Maximum Likelihood (LIML), Full Information Maximum Likelihood (FIML), and Indirect Least Squares (ILS). It was revealed that 2SLS, LIML and ILS estimators were the best for lower open interval negative level of multicollinearity while FIML and OLS were best for closed interval and upper categories level of multicollinearity.

Agunbiade (2011) investigated effect of multicollinearity and sensitivity of three estimators in three just identified simultaneous equation model with the aid of Monte Carlo approach. The study was estimates using mean of estimates and its bias. However, the study revealed that identical estimates as the assumed parameter was not produced but some estimates are quite close. The use of shapely value regression at the second stage of two-stage least squares for simultaneous equation when there is collinearity was also proposed by Mishra (2017). It was observed that all the structural coefficients estimated with the proposed two-stage least squares have an expected sign and can help to overcome the problem of collinearity.

A biased estimation method was proposed by Ozbay and Toker (2018) to remedy the problem of multicollinearity that exists in simultaneous equations model. Two-parameter estimation in linear regression model is carried out to the simultaneous equations model. Monte Carlo experiment and real life data were used to evaluate the proposed method. The performance of the estimation of the new method is better than the conventional two stage least squares estimator.

Many researchers have provided alternative solutions like M-estimator for outliers (see Brikes and Dodge (1993), Khan *et al.* (2021)) and ridge estimator for multicollinearity (Hoerl and Kennard, 1970). However, these methods cannot be applied to outlier and multicollinearity when they occur in the data at the same time (Jadhav and Kashid, 2011). No researchers have used Bayesian method for outlier and multicollinearity problems together in Simultaneous equation model. In this study, we consider the two problems together, that is, multicollinearity and outliers in Simultaneous equation model by using a Bayesian method of estimation. The performance of Bayesian method and some classical Simultaneous equation methods in the presence of outlier and multicollinearity will be compared to know their strengths.

The remainder of the work is organised as follows: Section 2 gives an overview of simultaneous equation model while Section 3 illustrates the Bayesian method for solving the multicollinearity and outlier problems in simultaneous equation model. Section 4 presents the design of the Monte Carlo experiment and results obtained from the experiment are presented and discussed in Section 5. Section 6 concludes.

1.2. Simultaneous equation model

Consider the following two structural equations of simultaneous model;

$$y_{1t}= \beta_{12}y_{2t}+\gamma_{11}x_{1t}+ \gamma_{12}x_{2t}+u_{1t} \qquad (1)$$

$$y_{2t}= \beta_{21}y_{1t}+\gamma_{22}x_{2t}+\gamma_{23}x_{3t}+u_{2t} \qquad (2)$$

where, y_{1t} and y_{2t} are the endogenous variables at time t and x_{1t} , x_{2t} and x_{3t} are the exogenous or predetermined variables. The u_{1t} and u_{2t} are the random disturbance terms assumed to be independently and identically normally distributed with zero means and finite variance-covariance matrix $\sum$ i.e $u \sim$ NID $(0, \sum)$. Also β_{12} , β_{21} , γ_{11}, γ_{12}, γ_{22} and γ_{23} are unknown population parameters of the model.

The simultaneous equations given in (1) and (2) can further also be written as:

$$y_{1t} = \beta_{12}y_{2t}+\gamma_{11}x_{1t}+ \gamma_{12}x_{2t}+0x_{3t}+u_{1t} \qquad (3)$$

$$y_{2t} = \beta_{21}y_{1t}+0x_{1t}+\gamma_{22}x_{2t}+\gamma_{23}x_{3t}+u_{2t} \qquad (4)$$

Rearranging, we have;

$$y_{1t}- \beta_{12}y_{2t}= \gamma_{11}x_{1t}+ \gamma_{12}x_{2t}+0x_{3t}+u_{1t} \qquad (5)$$

$$-\beta_{21}y_{1t}+ y_{2t}=0x_{1t}+\gamma_{22}x_{2t}+ \gamma_{23}x_{3t}+u_{2t} \qquad (6)$$

$$y=\begin{bmatrix} 1 & -\beta_{12} \\ -\beta_{21} & 1 \end{bmatrix}^{-1} \begin{bmatrix} \gamma_{11} & \gamma_{12} & 0 \\ 0 & \gamma_{22} & \gamma_{23} \end{bmatrix} \begin{bmatrix} x_{1t} \\ x_{2t} \\ x_{3t} \end{bmatrix} + \begin{bmatrix} 1 & -\beta_{12} \\ -\beta_{21} & 1 \end{bmatrix}^{-1} \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} \qquad (10)$$

In matrix form;

$$\begin{bmatrix} 1 & -\beta_{12} \\ -\beta_{21} & 1 \end{bmatrix} \begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} \gamma_{11} & \gamma_{12} & 0 \\ 0 & \gamma_{22} & \gamma_{23} \end{bmatrix} \begin{bmatrix} x_{1t} \\ x_{2t} \\ x_{3t} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} \qquad (7)$$

This can be written in reduced from as:

$$\beta y = \Gamma x + u$$

$$y=\beta^{-1}\Gamma x+\beta^{-1} u \qquad (8)$$

$$= \Pi x + v \qquad (9)$$

where $\beta = \begin{bmatrix} 1 & -\beta_{12} \\ -\beta_{21} & 1 \end{bmatrix}$,

$$\Gamma = \begin{bmatrix} \gamma_{11} & \gamma_{12} & 0 \\ 0 & \gamma_{22} & \gamma_{23} \end{bmatrix}$$

$$y = \begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix}$$

$$x = \begin{bmatrix} x_{1t} \\ x_{2t} \\ x_{3t} \end{bmatrix}$$

$$u = \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

$$y= \frac{1}{1-\beta_{12}\beta_{21}} \begin{bmatrix} 1 & \beta_{12} \\ \beta_{21} & 1 \end{bmatrix} \begin{bmatrix} \gamma_{11} & \gamma_{12} & 0 \\ 0 & \gamma_{22} & \gamma_{23} \end{bmatrix} \begin{bmatrix} x_{1t} \\ x_{2t} \\ x_{3t} \end{bmatrix} + \frac{1}{1-\beta_{12}\beta_{21}} \begin{bmatrix} 1 & \beta_{12} \\ \beta_{21} & 1 \end{bmatrix} \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} \qquad (11)$$

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \frac{1}{1-\beta_{12}\beta_{21}} \begin{bmatrix} \gamma_{11}x_{1t} + (\gamma_{12} + \beta_{12}\gamma_{22})x_{2t} + \beta_{12}\gamma_{23}x_{3t} \\ \beta_{21}\gamma_{11}x_{1t} + (\gamma_{22} + \beta_{21}\gamma_{12})x_{2t} + \gamma_{23}x_{3t} \end{bmatrix} + \begin{bmatrix} u_{1t} + \beta_{12}u_{2t} \\ \beta_{21}u_{1t} + u_{2t} \end{bmatrix} \qquad (12)$$

$$y_{1t} = \left[\frac{\gamma_{11}}{1-\beta_{12}\,\beta_{21}}\right] x_{1t} + \left[\frac{\gamma_{12}+\beta_{12}\,\gamma_{22}}{1-\beta_{12}\,\beta_{21}}\right] x_{2t} + \left[\frac{\beta_{12}\,\gamma_{23}}{1-\beta_{12}\beta_{21}}\right] x_{3t} + \left[\frac{1}{1-\beta_{12}\,\beta_{21}}\right] u_{1t} + \left[\frac{\beta_{12}}{1-\beta_{12}\,\beta_{21}}\right] u_{2t}$$

$$y_{2t} = \left[\frac{\beta_{21}\gamma_{11}}{1-\beta_{12}\,\beta_{21}}\right] x_{1t} + \left[\frac{\gamma_{22}+\beta_{21}\,\gamma_{12}}{1-\beta_{12}\,\beta_{21}}\right] x_{2t} + \left[\frac{\gamma_{23}}{1-\beta_{12}\beta_{21}}\right] x_{3t} + \left[\frac{\beta_{21}}{1-\beta_{12}\,\beta_{21}}\right] u_{1t} + \left[\frac{1}{1-\beta_{12}\,\beta_{21}}\right] u_{2t} \quad (13)$$

2.1 Bayesian method

The Bayesian method is based on Bayes theorem, where available knowledge about parameters in a statistical model is updated. It also provides a general approach for combining a modeller's beliefs with the evidence contained in the data Ojo (2021). Hence, Bayesian method entails three concepts namely; likelihood function, prior and posterior distribution.

2.2 Likelihood function

The likelihood function is the principal to the process of estimation of unknown parameters in Bayesian analysis. In Bayesian method, it is better to work on reduced form rather than the structural form due to prior elicitation and identification problem (see Dreze and Richard (1983), Bauwens (1984), and Kleibergen & Zivot (2003) for more details).

Recall from equation (9),

$$y = \Pi\, x + v$$

Using the definition of multivariate Normal Distribution, the likelihood can be written as:

$$P\,(D|\Pi,\Omega) = \frac{1}{(2\pi)\,|\Omega|^{N/2}} \exp\{-\tfrac{1}{2}\,(y - \Pi\, x)'\,\Omega^{-1}(y - \Pi\, x)\} \quad (14)$$

1. $P\,(D|\Pi,\Omega) = \frac{1}{(2\pi)\,|\Omega|^{N/2}}$ $\exp\{-\frac{1}{2}\, tr(M\,\Omega^{-1})\}$ (15)

where "tr" means the trace matrix, $|\Omega| = \det(\Omega)$, and $M = (r_{ij}) = (y - \Pi\, x)'(y - \Pi\, x)$, D is the given data.

2.3 Prior distribution

When there is absence of prior knowledge, using a non-informative prior in Bayesian inference can be of great value (see Datta and Ghosh, 1995, Kang, 2011 for more details on the use of non-informative prior). Here, we use a diffuse prior introduced by Jeffreys (1946), which is given as:

$$P(\Pi,\Omega) = P(\Pi)\,P(\Omega) \propto |\Omega|^{-3/2} \quad (16)$$

2.4 Posterior distribution

The posterior distribution summarizes what we know about uncertain quantities. It gathers all the evidence or information that has been taken into account by prior distribution. Hence, it combines both the likelihood and prior distribution.

Therefore, the joint posterior density is proportional to the likelihood times prior and can simply be written as:

$$P\,\Pi,\Omega|\,D) = |\Omega|\exp\{-\tfrac{1}{2}\,tr(M\,\Omega^{-1})\} \quad (17)$$

The major drawback of Bayesian method is that all joint posterior density of processes and parameters have to be specified via collection of conditional distributions.

Hence, the conditionals densities forms of (17) are given as:

$$P\,\Pi|\Omega,D) \sim N(\widehat{\Pi},\widehat{\Omega}_{\Pi}) \quad (18)$$
$$P\,(\Omega|\,\Pi,D) \sim IW(M,\mathrm{n}) \quad (19)$$

Equations in (18) and (19) are Normal and Inverse Wishart distributions.

In order to obtain the point estimate from the posterior density functions, the conditional posteriors given in equations (18) and (19) can then be solved numerically; these can be achieved by using the widely used method called

Markov Chain Monte Carlo (MCMC) (see Zellner and Min 1995, Percy, 1996 for MCMC description and applications).

3. Monte Carlo Experiment

In this Section, a Monte Carlo experiment will be setup to facilitate comparison between the Bayesian method and classical methods in simultaneous equation model that is characterized by multicollinearity and outliers. The steps of the experiment are outlined below:

3.1 Generate the exogenous variables x_{1t} , x_{2t} and x_{3t} for each sample point. Here, the variables will be generated from the uniform distribution (0, 1) Kmenta (1971) and Ojo and Adepoju (2013). These exogenous variables are characterized by multicollinearity and outliers. The levels of multicollinearity are:

3.1.2 High Multicollinearity (HM): 0.95 and 0.99

3.1.3 Low Multicollinearity (LM): 0.2 and 0.40 while the scenarios of outliers are: 0%, 10% and 20%.

- The initial values of the parameters are chosen arbitrarily given as: β_{21} = 0.5, β_{12} =0.8, γ_{11}=1.5, γ_{12}=1.0, γ_{22}=1.0, γ_{23}=1.0
- The disturbance terms, u_{1t} and u_{2t} will also be generated at each sample point.
- The disturbance terms and exogenous variables will be used to generate the endogenous variables.
- The sample sizes considered are 15, 50, and 100 while each of the samples is replicated10000 times, burn-in period=1000.

4. Results and Discussion

This Section discusses the results from the Monte Carlo experiment described in Section 3. Two criteria namely Absolute bias (ABIAS) and Mean squared error (MSE) will be used. The MSE and ABIAS for estimators namely; Bayesian, two stage least squares, three stage least squares, Limited information Maximum likelihood, and Ordinary least Squares are obtained for different sample sizes of collinearity and outliers in Tables 1 and 2. It was observed that the estimates of two stage least squares, three stage least squares, and Limited information Maximum likelihood are the same, hence they are represented by 23LIML while Bayesian is represented by Bayes. The estimator with the minimum ABIAS and MSE is the most efficient.

From Table 1, the Bayes method has the least absolute bias followed by 23SLS while the OLS method has the largest absolute bias for the two equations for all the levels of collinearity. It was also observed that the bias estimates decreases the sample sizes increases for all the methods considered across the levels of collinearity. The ABIAS obtained in both equations 1 and 2 for 10% and 20% are higher than when there no outlier. The ABIAS estimates for equation 1 are smaller than equation 2 for all the sample sizes considered across both levels of outliers and collinearity.

In Table 2, it is observed Bayes method gives the minimum MSE for the entire sample sizes considered followed by 23LIML while OLS has the highest MSE. All the methods are not greatly affected by outliers; however when the percentage of contamination goes to 10% and 20%, the MSE of the estimators increases. For low level of collinearity, the MSE are minimal.

5. Conclusion

Multicollinearity and outliers are great problems in applied work. This work determined the best method of estimation when a just identified simultaneous equation model has both problem of multicollinearity and outliers. The method considered were Bayesian, two stage least squares; three stage least squares, limited information maximum likelihood, and ordinary least squares. The performance of the estimators were evaluated using mean square error and absolute bias. When there is no outliers, all the estimators have the same performances, however when the levels of outlier were 10% and20%, the estimates of the estimators increases. The Absolute bias and mean squared error estimates of the estimators increases as the level of collinearity also increases. Also, all the methods considered show consistent asymptotic pattern with values of absolute bias and Mean squared error decreasing consistently. Bayesian method of estimation outperformed all other method of estimation when mean square error and absolute bias were used as criteria. Hence, Bayesian method of estimation is considered the best estimator when a just identified simultaneous equation model has both problem of multicollinearity and outliers.

6. Code availability

The code used can be obtained from the corresponding author.

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Appendix

Table 1: Absolute Bias of the estimators with varying sample sizes for collinearity and outliers

Sample size			15			50			100		
Eqn	Method	Collinearity levels	Outliers			Outliers			Outliers		
			0	10	20	0	10	20	0	10	20
1	OLS	0.99	2.056	2..353	2.436	1.665	1.814	1.921	1.451	1.513	1.613
		0.60	0.714	0.913	1.274	0.501	0.610	0.736	0.247	0.426	0.510
		0.20	0.561	0.610	0.402	0.492	0.592	0.720	0.158	0.013	0.218
	23LIML	0.99	1.583	1.393	1.691	1.329	1.489	1.391	0.481	0.821	0.942
		0.60	0.492	0.691	1.024	0.392	0.529	0.492	0.192	0.291	0.482
		0.20	0.329	0.492	0.692	0.382	0.182	0.321	0.018	0.128	0.181
	Bayes	0.99	0.197	0.192	1.283	0.732	0.821	0.913	0.002	0.148	0.285
		0.60	0.219	0.271	0.294	0.031	0.059	0.071	0.001	0.081	0.103
		0.20	0.004	0.019	0.028	0.008	0.004	0.017	0.001	0.019	0.027
2	OLS	0.99	5.356	6.153	6.913	4.414	4.810	5.012	3.014	4.028	4.821
		0.60	4.829	4.914	5.018	3.091	4.191	4.391	2.018	2.918	3.048
		0.20	3.829	3.991	4.192	3.012	3.291	4.014	1.041	2.492	2.563
	23LIML	0.99	0.356	0.356	0.356	0.465	0.821	0.917	0.618	0.7183	1.632
		0.60	0.271	0.483	0.282	1.593	1.829	2.491	0.392	0.219	0.192
		0.20	0.282	0.193	0.493	0.192	1.452	1.823	0.138	0.319	0.218
	Bayes	0.99	0.008	0.013	0.210	0.013	0.029	0.193	0.001	0.021	0.043
		0.60	0.142	0.004	0.103	0.002	0.081	0.093	0.020	0.028	0.033
		0.20	0.010	0.029	0.031	0.028	0.033	0.076	0.001	0.001	0.001

Table 2: MSE of the estimators with varying sample sizes for collinearity and outliers

Sample size			15			50			100		
Eqn	**Method**	**Collinearity levels**	**Outliers**			**Outliers**			**Outliers**		
			0	**10**	**20**	**0**	**10**	**20**	**0**	**10**	**20**
1	**OLS**	**0.99**	6.192	7.396	7.921	4.019	5.829	6.183	2.029	2.712	2.098
		0.60	4.193	5.940	6.011	3.983	3.391	3.017	1.393	2.001	3.191
		0.20	5.193	5.812	5.889	2.001	2.397	4.191	1.430	1.552	2.083
	23LIML	**0.99**	4.289	4.908	4.716	1.933	4.882	5.018	0.255	1.380	0.829
		0.60	2.392	3.110	3.814	2.910	2.816	2.914	0.133	1.231	0.383
		0.20	2.135	3.022	3.007	2.136	2.490	3.025	0.582	0.800	1.227
	Bayes	**0.99**	1.669	1.382	1.888	1.216	1.305	1.529	0.501	0.628	0.723
		0.60	1.302	1.811	1.906	1.237	2.192	1.724	0.021	1.150	0.158
		0.20	0.628	2.820	2.977	2.243	1.428	2.518	0.281	0.778	0.993
2	**OLS**	**0.99**	8.160	8.936	7.522	7.461	7.722	4.206	4.119	4.296	2.911
		0.60	5.993	7.200	6.293	3.916	5.229	3.193	2.104	3.888	1.948
		0.20	3.813	4.001	4.729	2.917	3.914	4.285	1.395	1.732	0.813
	23LIML	**0.99**	4.359	0.356	0.356	3.465	4.028	3.281	2.001	2.875	1.377
		0.60	3.006	4.913	0.182	1.110	2.913	2.015	1.393	1.118	0.724
		0.20	1.842	3.927	1.996	2.537	2.114	2.439	0.832	1.027	1.279
	Bayes	**0.99**	0.629	0.669	0.703	0.518	0.592	0.551	0.281	0.319	0.402
		0.60	0.382	0.490	0.511	0.317	0.423	0.518	0.201	0.289	0.318
		0.20	0.029	0.201	0.388	0.026	0.173	0.192	0.000	0.001	0.016