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# Bound State Solutions of the Schrodinger's Equation with Manning-Rosen plus Yukawa Potential using Pekeris-like Approximation of the Coulomb Term and Parametric Nikoforov-Uvarov

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## ABSTRACT

The solutions of the Schrödinger equation with Manning-Rosen plus Yukawa potential (MRYP) have been presented using the Pekeris-like approximation of the coulomb term and parametric Nikiforov-Uvarov (NU) method. The bound state energy eigenvalues and the corresponding un-normalized eigen functions are obtained in terms of Jacobi polynomials. Also, Yukawa, Manning-Rosen and coulomb potentials have been recovered from the mixed potential and their eigen values obtained.The Numerical results are computed for some values of n at l=0 with α = 0.01, 0.1, 2 and 5 using python 3.6 programming, and these results could be applied to molecules moving under the influence of MRYP potential as negative energy eigenvalues obtained indicate a bound state system.

**Keywords**: Schrödinger equation, Manning-Rosen potential, Yukawa potential,Pekeris-like approximation, ParametricNikiforov-Uvarov method, Jacobi polynomials

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## 1 INTRODUCTION

In quantum mechanics, one of the interesting problems is to obtain exact solutions of the Schrödinger equation. In order to do this, a real potential is normally chosen to derive the energy eigenvalues and the eigen functions of the Schrödinger equation.[1] These solutions describe the particle dynamics in non-relativistic quantum mechanics.[2] Several authors have studied the bound states of the Schrödinger equation using different potentials and methods. Some of these potentials play very important roles in many fields of Physics such as Molecular Physics, Solid State and Chemical Physics.[3] The Manning-Rosen potential has been deeply studied and applied in quantum systems and Yukawa potential and its classes have been studied in Schrodinger formalism.[4]

The purpose of the present paper is to solve the Schrödinger equation for the mixed potential MRYP using the parametric NU method. The paper is organized as follows: After a brief introduction in section 1, the NU method is reviewed in section 2. In section 3, we solve the radial Schrödinger equation using the NU method. Finally, we discuss our results in section

4 and a brief conclusion is then advanced in section 5.

## 2 NIKIFOROV-UVAROV METHOD

The Nikiforov-Uvarov (NU) method[5] is based on the solutions of a generalized second-order linear differential equation with special orthogonal functions. The Schrödinger equation and Schrödinger-like equations of the type as:

$$\psi''(r) + [E - V(r)]\psi(r) = 0, \qquad (1)$$

can be solved by this method. To do this equation (1) is transformed into equation of hypergeometric type with appropriate coordinate transformation $s = s(r)$ to get

$$\psi''(s) + \frac{\bar{\tau}(s)}{\sigma(s)}\psi'(s) + \frac{\bar{\sigma}(s)}{\sigma^2(s)}\psi(s) = 0, \qquad (2)$$

To solve equation (2) we can use the parametric NU method. The parametric generalization of the NU method is expressed by the generalized hypergeometric type equation [19]

$$\psi''(s) + \frac{(c_1 - c_2 s)}{s(1 - c_3 s)}\psi'(s) + \frac{1}{s^2(1 - c_3 s)^2}[-\epsilon_1 s^2 + \epsilon_2 s - \epsilon_3]\psi(s) = 0, \qquad (3)$$

where $\sigma(s)$ and $\bar{\sigma}(s)$ are polynomials at most second degree, and $\bar{\tau}(s)$ is a first degree polynomial. The eigenfunctions (equation 4) and corresponding eigenvalues (equation 5) to the equation become

$$\psi(s) = N_n s^{c_{12}}(1 - c_3 s)^{-c_{12} - \frac{c_{13}}{c_3}} P_n^{\left(c_{10} - 1, \frac{c_{11}}{c_3} - c_{10} - 1\right)}(1 - 2c_3 s), \qquad (4)$$

$$(c_2 - c_3)n + c_3 n^2 - (2n + 1)c_5 + (2n + 1)\left(\sqrt{c_9} + c_3\sqrt{c_8}\right) + c_7 + 2c_3 c_8 + 2\sqrt{c_8 c_9} = 0, \qquad (5)$$

Where

$$c_4 = \frac{1}{2}(1 - c_1), c_5 = \frac{1}{2}(c_2 - 2c_3), c_6 = {c_5}^2 + \epsilon_1, c_7 = 2c_4 c_5 - \epsilon_2, c_8 = {c_4}^2 + \epsilon_3, \quad c_9 = c_3 c_7 + {c_2}^2 c_8 + c_6, \quad c_{10} = c_1 + 2c_4 + 2\sqrt{c_8}, c_{11} = c_2 - 2c_5 + 2\left(\sqrt{c_9} + c_3\sqrt{c_8}\right), c_{12} = c_4 + \sqrt{c_8},$$
$$c_{13} = c_5 - \left(\sqrt{c_9} + c_3\sqrt{c_8}\right), \qquad (6)$$

$N_n$ is the normalization constant and ${P_n}^{(\alpha,\beta)}$ are the Jacobi polynomials.

## 3. SOLUTIONS OF THE RADIAL PART OF SCHRÖDINGER EQUATION WITH MRYP POTENTIAL

The radial Schrödinger equation[6] is given as

$$\frac{d^2 R_{nl}(r)}{dr^2} + \frac{2\mu}{\hbar^2}\left[E - V(r) - \frac{\lambda\hbar^2}{2\mu r^2}\right]R_{nl}(r), \qquad (7)$$

Where $\lambda = l(l+1)$ and $V(r)$ is the potential energy function. The Manning-Rosen potential (MRP) is given as:

$$V(r) = -\left[\frac{Ce^{-\propto r} + De^{-2\propto r}}{(1 - e^{-\propto r})^2}\right] \qquad (8)$$

The Yukawa potential (YP) is given as:

$$V(r) = -\frac{V_o e^{-\propto r}}{r}, \qquad (9)$$

where $V_0$ is the potential depth of the YP and $\alpha$ is an adjustable positive parameter. In equation (8) $C$ and $D$ are constants. The sum of these potentials known as MRYP is given as

$$V(r) = -\left[\frac{Ce^{-\propto r} + De^{-2\propto r}}{(1 - e^{-\propto r})^2}\right] - \frac{V_0 e^{-\propto r}}{r} \qquad (10)$$

Making the transformation $s = e^{-\alpha r}$ equation (10) becomes

$$V(s) = -\left[\frac{CS + DS^2}{(1 - S)^2}\right] - \frac{\propto V_0 S}{1 - S} \qquad (11)$$

Again, applying the transformation $s = e^{-\alpha r}$ and using the Pekeris-type approximation[7], we get the form that NU method is applicable, equation (7) gives a generalized hypergeometric-type equation as

$$\frac{d^2 R(s)}{ds^2} + \frac{(1 - s)}{(1 - s)s}\frac{dR(s)}{ds} + \frac{1}{(1 - s)^2 s^2}[-(\beta^2 - F + B)s^2 + (2\beta^2 + A + B)s - (\beta^2)]R(s) = 0, \qquad (12)$$

Where $\lambda = 0, -\beta^2 = \frac{2\mu E}{\alpha^2 \hbar^2}, A = \frac{2\mu C}{\alpha^2 \hbar^2}, B = \frac{2\mu V_0}{\alpha \hbar^2}, F = \frac{2\mu D}{\alpha^2 \hbar^2}, \frac{1}{r} \approx \frac{\propto}{(1-e^{-\propto r})} \approx \frac{\propto}{(1-S)},$ (13)

Comparing equation (12) with equation (3) yields the following parameters

$$c_1 = c_2 = c_3 = 1, c_4 = 0, c_5 = -\frac{1}{2}, c_6 = \frac{1}{4} + \beta^2 + B - F, c_7 = -2\beta^2 - A - B, c_8 = \beta^2, c_9 = \frac{1}{4} - (A + F), c_{10} = 1 + 2\sqrt{\beta^2}, c_{11} = 2 + 2\left(\sqrt{\frac{1}{4} - A - F} + \sqrt{\beta^2}\right), c_{12} = \sqrt{\beta^2}, c_{13} = -\frac{1}{2} - \left(\sqrt{\frac{1}{4} - A - F} + \sqrt{\beta^2}\right), \epsilon_1 = \beta^2 + B - F, \epsilon_2 = 2\beta^2 + A + B, \epsilon_3 = \beta^2, \quad (14)$$

Now using equations (5), (13) and (14) we obtain the energy eigen spectrum of the MRYP as

$$\beta^2 = \left[\frac{A+B-\left(n^2+n+\frac{1}{2}\right)-(2n+1)\sqrt{\frac{1}{4}-A-F}}{(2n+1)+2\sqrt{\frac{1}{4}-A-F}}\right]^2, \quad (15)$$

Equation (15) can be solved explicitly and the energy eigen spectrum of MRYP becomes

$$E = -\frac{\alpha^2\hbar^2}{2\mu}\left\{\left[\frac{\frac{2\mu C}{\alpha^2\hbar^2}+\frac{2\mu V_0}{\alpha\hbar^2}-\left(n^2+n+\frac{1}{2}\right)-(2n+1)\sqrt{\frac{1}{4}-\frac{2\mu C}{\alpha^2\hbar^2}-\frac{2\mu D}{\alpha^2\hbar^2}}}{(2n+1)+2\sqrt{\frac{1}{4}-\frac{2\mu C}{\alpha^2\hbar^2}-\frac{2\mu D}{\alpha^2\hbar^2}}}\right]^2\right\} \quad (16)$$

We now calculate the radial wave function of the MRYP as follows

The weight function $\rho(s)$ is given as [19]

$$\rho(s) = s^{c_{10}-1}(1 - c_3 s)^{\frac{c_{11}}{c_3} - c_{10} - 1}, \quad (17)$$

Using equation (14) we get the weight function as

$$\rho(s) = s^U (1 - s)^V, \quad (18)$$

Where $U = 2\sqrt{\beta^2}$ and $V = 2\sqrt{\frac{1}{4} - A - F}$

Also we obtain the wave function $\chi(s)$ as [19]

$$\chi(s) = P_n^{c_{10}-1, \frac{c_{11}}{c_3} - c_{10} - 1}(1 - 2c_3 s), \quad (19)$$

Using equation (14) we get the function χ(s) as

$$\chi(s) = P_n^{(U,V)}(1 - 2s), \quad (20)$$

Where $P_n^{(U,V)}$ are Jacobi polynomials

Lastly,

$$\varphi(s) = s^{c_{12}}(1 - c_3 s)^{-c_{12} - \frac{c_{13}}{c_3}}, \quad (21)$$

And using equation (14) we get

$$\varphi(s) = s^{U/2}(1 - s)^{V - 1/2}, \quad (22)$$

We then obtain the radial wave function from the equation [19]

$$R_n(s) = N_n \varphi(s) \chi_n(s), \quad (23)$$

As

$$R_n(s) = N_n s^{U/2}(1 - s)^{(V-1)/2} P_n^{(U,V)}(1 - 2s), \quad (24)$$

Where $n$ is a positive integer and $N_n$ is the normalization constant.

## 4 DISCUSSION:

We have solved the radial Schrödinger equation and obtained the energy eigen values for the Manning-Rosen plus Yukawa potential (MRYP) in equation (16). We calculated the bound state energy eigenvalues by variation of the screening parameter as shown in Table 1.

**Table 1. Energy eigenvalues $E(eV)$ of the MRYP potential for ђ=μ=1, $V_0 = 0.2$, C = -0.1, D = 0.1 with different α values.**

| $n$ | $\alpha = 0.01$ | $\alpha = 0.1$ | $\alpha = 2$ | $\alpha = 5$ |
|---|---|---|---|---|
| 1 | $-12.0540$ | $-0.124999$ | $-1.852812$ | $-12.05405$ |
| 2 | $-5.38466$ | $-0.08680$ | $-4.35124$ | $-27.67680$ |
| 3 | $-3.05045$ | $-0.08$ | $-7.85070$ | $-49.55101$ |
| 4 | $-1.97011$ | $-0.08405$ | $-12.35045$ | $-77.67564$ |
| 5 | $-1.38333$ | $-0.093889$ | $-17.85031$ | $-112.0504$ |
| 6 | $-1.02961$ | $-0.10778$ | $-24.3502$ | $-152.6753$ |
| 7 | $-0.80011$ | $-0.125000$ | $-31.8501$ | $-199.5502$ |

The following cases are considered:

Case 1: If $C = D = 0$ in equation (10), the potential turns back into the Yukawa potential and equation (16) yields the energy eigen values of the Yukawa potential as

$$E = -\frac{\hbar^2}{2\mu}\left[\frac{\frac{2\mu V_0}{\hbar^2} - \alpha^2(n+1)^2}{2(n+1)}\right]^2, \qquad (25)$$

Equation (25) is similar to equation (30) obtained by Antia *et al*., 2015

Case 2: If $\alpha \to 0, V_0 = Ze^2$ in equation (25), the energy eigen values for Coulomb potential becomes

$$\mathrm{E} = -\frac{Z^2 e^4 \mu}{2\hbar^2 n'^2} \qquad (26)$$

Where $n' = n + 1$ in this case.

Case 3: If $V_0 = 0$ the potential in equation (10) yields the Manning-Rosen potential with energy eigen values given as

Eq. (27) is also similar to Manning-Rosen potential bound state energy obtained by Louis *et al*.[4]

$$E = -\frac{\alpha^2\hbar^2}{2\mu}\left\{\left[\frac{\frac{2\mu C}{\alpha^2\hbar^2} - \left(n^2+n+\frac{1}{2}\right) - (2n+1)\sqrt{\frac{1}{4} - \frac{2\mu C}{\alpha^2\hbar^2} - \frac{2\mu D}{\alpha^2\hbar^2}}}{(2n+1) + 2\sqrt{\frac{1}{4} - \frac{2\mu C}{\alpha^2\hbar^2} - \frac{2\mu D}{\alpha^2\hbar^2}}}\right]^2\right\} \qquad (27)$$

**5. CONCLUSION**:

We have obtained the energy eigen values and the corresponding un-normalized wave function using the parametric NU method for the Schrӧdinger equation with MRYP. Special cases of the potential have also been considered.

## References

Antia, A.D, Essien, E. B Umoren and C.C Eze, (2015) Approximate Solution of the non-relativistic Schrodinger Equation with Inversely Quadratic Yukawa Plus Mobius Square Potential Via Parametric Nikiforov-Uvarov Method, Advances in Physics Theories and Application, Vol. 44, pp. 38-41

Ita, B.I, A.I Ikeuba, O. Obinna, (2015). Solution of Schrödinger Equation with Inversely Quadratic Yukawa potential plus Woods-Saxon potential using parametric Nikiforov-Uvarov method, Journals of Advances in Physics 18(2), pp. 2094-2098.

Ita, B.I, Nyong, N.O. Alobi, H. Louis and T.O. Magu**:** Bound State S Gordon
Equation for Modified Echart Plus Inverse Square Molecular Potential with Improved
new Approximation Scheme to Centrifugal Team. Equatorial Journal olution of the Klein- Gordon Equation for Modified Echart Plus Inverse Square Molecular Potential
Computational and Theoretical Sciemces, Volume 1, Issue 1, (2016), pp. 55-64.

Louis, H, Ita, B.E Nyong, T.O Magu, N.A Nzeata-ibe and S. Barka, "Radial Solution of the s-wave D-Dimensional Non-relativistic Schrodinger equation for Generalized Manning-Rosen plus Mie-type Nuclei potential wirthin the framework of Nikiforov-Uvarov Method" J. of NAMP., Vol. 36, No. 2, pp 193-198 (2016)

Magu, T.O. Ita, B.I, Nyong, B.E, H. Louis: Radial solution of the s-wave Klein-Gordon equation for generalied wood-saxon plus Mie-type potential using Nikifarov-Uvarov J. Chem. Soc. Nigeria, Vol. 41, No. 2, pp21-26 [2017].